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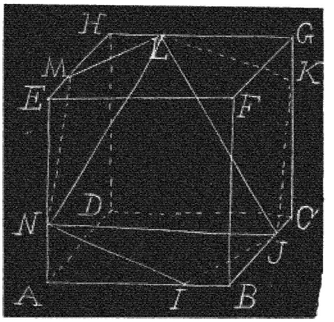
AVERAGE AND PROBABILITY.

113. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy, Defiance College, Defiance. Ohio.

A given cube is cut by a plane in such a manner that the lines of section form a regular hexagon. What is the mean area of this hexagon?

Solution by G. B. M. ZERR. A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.



Let $ABCD-H$ be the cube, side a.

$HM=EN=AI=BJ=CK=GL=x$.

$LH=EM=AN=BI=CJ=GK=a-x$.

Then $IJKLMN$ is a hexagon.

$IJ=JK=KL=LM=MN=NI=\sqrt{(a^2+2x^2-2ax)}$.

$IK=JL=KM=LN=MI=NJ=\sqrt{(2a^2+2x^2-2ax)}$.

$\therefore$ The hexagon is a regular hexagon.

Area of this hexagon $=\frac{3}{2}(a^2+2x^2-2ax)\sqrt{3}=u$.

Average area $=\triangle=\int_{\frac{1}{2}a}^{a}udx\Big/\int_{\frac{1}{2}a}^{a}dx$.

$$\therefore \triangle=\frac{3\sqrt{3}}{a}\int_{\frac{1}{2}a}^{a}(a^2+2x^2-2ax)dx=a^2\sqrt{3}.$$

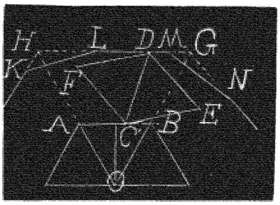
114. Proposed by L. C. WALKER, A. M.. Professor of Mathematics, Petaluma High School, Petaluma, Cal.

If a regular polygon of n sides be placed at random on another equal polygon, show that the chance that the center of the first will fall on the second polygon is $\dfrac{\pi}{2[\pi+n\tan(\pi/n)]}$.

Solution by G. B. M. ZERR, A. M.. Ph. D.. Professor of Chemistry and Physics in The Temple College, Philadelphia. Pa.

Let AOB, DCE be a triangle of each polygon. Let DCE move parallel to itself, then the center D will trace out the polygon of $2n$ sides $KLMGN$, etc.

Let $BO=CD=r$, $\angle BCE=\theta=$the angle BO makes with DC. Then the perpendicular distance from AB to $HG=r\cos[(\pi/n)-\theta]$, $HA=r\sec(\pi/n)\cos[(\pi/n)-\theta]=r[\cos\theta+\sin\theta\tan(\pi/n)]$, $OH=r[1+\cos\theta+\sin\theta\tan(\pi/n)]$.



$MG=\sqrt{(CD^2+BG^2-2CD.BG\cos\theta)}=r\sin\theta\sec(\pi/n)$.

$HG=2r[1+\cos\theta+\sin\theta\tan(\pi/n)]\sin(\pi/n)$.

$HL=HG-LG=r[2\cos\theta\sin(\pi/n)+2\sin\theta\sin(\pi/n)\tan(\pi/n)-\sin\theta\sec(\pi/n)]$.

Area $HOG=\frac{1}{2}r^2[1+\cos\theta+\sin\theta\tan(\pi/n)]^2\sin(2\pi/n)$.

Area $MGN=\frac{1}{2}r^2[2\sin\theta\cos\theta\tan(\pi/n)+\sin^2\theta\tan^2(\pi/n)-\sin^2\theta]\sin^2(\pi/n)$.

The total number of positions is $n(\triangle OGH-\triangle MGN)=nr^2\sin(2\pi/n)[1+\cos\theta+\sin\theta\tan(\pi/n)]=A$. The number of favorable positions is $n\triangle AOB=B$.

$\therefore B=\frac{1}{2}nr^2\sin(2\pi/n)$.

$$\therefore p=\int_0^{\pi/n}\beta d\theta\Big/\int_0^{\pi/n}Ad\theta=\tfrac{1}{2}\int_0^{\pi/n}d\theta\Big/\int_0^{\pi/n}[1+\cos\theta+\sin\theta\tan(\pi/n)]d\theta=\frac{\pi}{2[\pi+n\tan(\pi/n)]}$$

COR. When $n=\infty$, $p=\frac{1}{4}$; when $n=3$, $p=\pi/[2(\pi+3\sqrt{3})]$; when $\pi=4$, $p=\pi/[2(\pi+4)]$; when $n=6$, $p=\pi/[2(\pi+2\sqrt{3})]$.